# Hadron-nucleon Total Cross Section Fluctuations from Hadron-nucleus Total Cross Sections

David R. Harrington

*Department of Physics and Astronomy, Rutgers University*

*P.O. Box 849, Piscataway, NJ 08855-0849 USA*

(February 9, 2008)

## Abstract

The extent to which information about fluctuations in hadron-nucleon total cross sections in the frozen approximation can be extracted from very high energy hadron-nucleus total cross section measurements for a range of heavy nuclei is discussed. The corrections to the predictions of Glauber theory due to these fluctuations are calculated for several models for the distribution functions, and differences of the order of 50 mb are found for heavy nuclei. The generating function for the moments of the hadron-nucleon cross section distributions can be approximately determined from the derivatives of the hadron-nucleus total cross sections with respect to the nuclear geometric cross section. The argument of the generating function, however, it limited to the maximum value of a dimensionless thickness function obtained at zero impact parameter for the heaviest nuclear targets: about 1.8 for pions and 3.0 for nucleons.

11.80.La, 13.85Lg, 25.40.Cm, 25.40 Dn, 25.80.Dj

Typeset using REVTEX

# I. INTRODUCTION

There has recently been a revival of interest in the use of the cross section distribution function to describe certain features of hadronic collisions [1–3]. The general approach, which goes back to the work of Good and Walker [4] in 1960, begins with the composite nature of hadrons and assumes that at high energies the internal degrees of freedom are "frozen" during the collision [5]. Many features of high energy reactions can then be discussed, at least approximately, in terms of a single distribution function which gives the probability that the initial hadron is in one of the configurations which interacts with a given total cross section. Discussions of this distribution function for incident pions and nucleons interacting with nucleons, including estimates of the 2nd and 3rd moments, are given in Refs. [1] and [2]. The discussion below tries to answer the question of whether more information about these distributions, and thus more information about the composite nature of the hadrons, can be obtained by a careful study of hadron-nucleus total cross sections.

The answer to this question is closely related to the importance of inelastic intermediate states in hadron-nucleus scattering, which has been discussed extensively [3]. In particular the systematic experimental study of Murthy *et al.* [6] confirmed earlier theoretical estimates of these effects, including their energy dependence. Here a similar discussion is presented in the language of the cross section distribution function. This approach is valid only in the high energy limit, where the frozen approximation has a chance of being valid, but has the advantage of including all orders of inelastic scattering, which might well be important for heavy nuclei.

Section II gives a brief review of how the distribution function is defined, and what is known about it. Then Sec. III summarizes the approximations and assumptions which are necessary to obtain a simple formula for the total hadron-nucleus cross section in terms of the generating function for the reduced distribution function, the argument being a reduced nuclear thickness function. In Sec. IV the limiting case of uniform nuclear density is discussed. It is shown that in this case the generating function of the distribution function is

just the derivative of the total cross section with respect to twice the cross sectional area of the nucleus. This result shows clearly that the amount of information we can hope to obtain from nuclear cross sections is limited because the argument of the generating function is proportional to the nuclear thickness function, and thus limited by the limited sizes of stable nuclei. This discussion is extended to the more realistic case of Woods-Saxon nuclear densities in Sec. V. For these densities the derivatives mentioned seem to approach the generating functions from above for large nuclei. In addition the numerical results of this section give some idea of the sensitivity of these quantities to reasonable changes in the distribution function, and thus an estimate of the experimental accuracy required to distinguish among different possibilities. These results are discussed in Sec. VI, along with suggestions for further investigations in this area.

## II. CROSS SECTION DISTRIBUTION FUNCTION

At high energy it is assumed that the internal configuration, here labeled by a Greek letter $\alpha$, $\beta$, etc., of a hadron interacting with a target is "frozen" during the interaction so that the transition amplitudes are diagonal in these states:

$$\langle \beta \mid F_{op} \mid \alpha \rangle \;=\; \delta_{\beta\alpha} F_{\alpha}, \tag{1}$$

where $F_{op}$ is the transition operator in the space of the internal degrees of freedom of the incident hadron. The amplitudes for transitions between ordinary hadronic mass eigenstates $i$ and $j$ on a given target are then

$$\langle j \mid F_{op} \mid i \rangle \;=\; \sum_{\alpha} \langle j \mid \alpha \rangle F_{\alpha} \langle \alpha \mid i \rangle, \tag{2}$$

where $i$ and $j$ must represent states with the same flavor quantum numbers, so $j$ must be either $i$ or a state which can be diffractively excited from it. Using the same normalization as Blättel $et$ $al.$ [1,2] the total cross section for hadron $i$ incident on the target is

$$\sigma_i \;=\; 4\pi Im\langle i \mid F_{op} \mid i \rangle \;=\; \sum_{\alpha} \sigma_{\alpha} \mid \langle \alpha \mid i \rangle \mid^{2}, \tag{3}$$

3

where $\sigma_\alpha$ is the total cross section for the hadron in configuration $\alpha$ to interact with the target.

If the cross section distribution function $P_i$ is defined as

$$P_i(\sigma) = \sum_\alpha | \langle \alpha \mid i \rangle |^2 \delta (\sigma - \sigma_\alpha), \tag{4}$$

this becomes

$$\sigma_i = \langle i \mid \sigma_{op} \mid i \rangle = \int P_i(\sigma) \, \sigma \, d\sigma, \tag{5}$$

i.e. $\sigma_i$ is just the average of $\sigma$ over the distribution $P_i$. The same distribution can also be used to calculate the forward total differential cross section for diffractive scattering, if the transition amplitudes are assumed to be pure imaginary, for then

$$\frac{d\sigma_{\text{diff}}}{dt} \Big|_{t=0} = \pi \sum_{j \neq i} | \langle j \mid F_{op} \mid i \rangle |^2$$
$$= (\langle \sigma^2 \rangle_i - \langle \sigma \rangle_i^2) / (16\pi). \tag{6}$$

For scattering in the high energy regime discussed by Blättel *et al.* [1,2], $\langle \sigma \rangle \approx 24$mb for pions on nucleons while $\langle \sigma \rangle \approx 40$mb for nucleons on nucleons. Experimental results on high energy forward diffraction dissociation on nucleons gives $\langle \sigma^2 \rangle \approx 1.25 \langle \sigma \rangle^2$ for nucleons, while $\langle \sigma^2 \rangle \approx (1.4 - 1.5) \langle \sigma \rangle^2$ for pions. It is also possible to put constraints on the 3rd moments of the distributions using diffraction dissociation on deuterons, but only by making fairly strong assumptions about the transitions between excited states.

For the developments below it is convenient to introduce reduced distribution functions

$$f_i(x) \equiv \langle \sigma \rangle \, P_i(\langle \sigma \rangle \, x), \tag{7}$$

so that

$$\langle x^n \rangle_i \equiv \int dx \, f_i(x) \, x^n = \langle \sigma^n \rangle_i / \langle \sigma \rangle_i^n. \tag{8}$$

The simple Poisson-like distributions

$$f_n(x) = [(n+1)^{n+1} / n!] x^n \exp [-(n+1)x] \tag{9}$$

4

will be used below for illustrative purposes because they lead to analytic formulas in some cases. The generating function, for example, is

$$\langle \exp\left(-xt\right)\rangle_n = \left[\frac{(n+1)}{(n+1+t)}\right]^{n+1},\tag{10}$$

leading to the moments

$$\langle x^m\rangle_n = \frac{(n+m)\,!}{n!\,(n+m)^m}\,.\tag{11}$$

Blättel *et al.* [1,2] suggest several different forms for the pion-nucleon and nucleon-nucleon cross section distribution functions, all of which fall off with $x$ more rapidly than the $f_n$'s. Based on the expected behavior of the pion and nucleon internal wave functions they also assume that f(0) vanishes for nucleons, but not for pions . The behavior at large $x$, however, is more problematic. The main object of this paper is to see if a careful and systematic study of hadron-nucleus total cross section can shed any new light on these distribution functions.

## III. HADRON-NUCLEUS CROSS SECTIONS

In Glauber theory, which itself uses the frozen approximation for the nucleons in the nucleus, the total cross section for scattering of a hadron $i$ from a nucleus A can be written as

$$\sigma_i(A) = 2\int d^2b\Gamma_i(A,b),\tag{12}$$

where

$$\Gamma_i(A,b) = 1 - \langle A\mid \Pi_\alpha\left(1 - \gamma_{i,\alpha}(b)\right)\mid A\rangle,\tag{13}$$

with $\gamma_{\mathrm{i},\alpha}$ the analogue of $\Gamma$ for the scattering of the hadron $i$ from the $\alpha$th nucleon in the nucleus. In "standard" Glauber theory, where the internal degrees of freedom of the hadron are ignored, $\gamma_{\mathrm{i},\alpha}$ is just a numerical function of $\mathbf{b}$ and $\mathbf{r}_\alpha$, the position of the $\alpha$th nucleon (except for possible spin- and isospin-dependence, which we ignore here). In general, however, $\gamma$ should be interpreted as an operator in the internal space of the hadron, and

$$\Gamma_i(A, b) = 1 - \langle A \mid \langle i \mid \Pi_\alpha \left(1 - \gamma_{\alpha op}(b)\right) \mid i \rangle \mid A \rangle. \tag{14}$$

This expression can be simplified considerably if a number of assumptions are made. First, if the nuclear wave function is completely uncorrelated

$$\Gamma_i(A, b) = 1 - \langle i \mid [1 - \langle A \mid \gamma_{1op}(b) \mid A \rangle]^A \mid i \rangle. \tag{15}$$

Then, if the range of $\gamma$ is much less than the nuclear size,

$$\Gamma_i(A, b) = 1 - \langle i \mid [1 - \frac{\sigma_{op}}{2} T(b)]^A \mid i \rangle, \tag{16}$$

where T(b), the thickness function, is the integral of the nuclear density along a straight line at impact parameter b, and $\sigma_{op}$ is twice the integral of $\gamma_{op}$ over impact parameter space. In general $\gamma_{op}$ and $\sigma_{op}$ are both operators in the internal space of the hadron taking complex values, but if the imaginary part is ignored then $\sigma_{op}$ is exactly the operator described by the distribution function $P_i$ introduced in Sec. II. Finally, if A is large and $\sigma_{op}T(b)$ takes only small values, then

$$\begin{aligned} \Gamma_i (A, b) &= 1 - \langle i \mid \exp \left[-\frac{\sigma_{op}}{2} T (b) A\right] \mid i \rangle \\ &= 1 - \langle i \mid \exp \left[-t (A, b) x_{op}\right] \mid i \rangle, \end{aligned} \tag{17}$$

where

$$t(A, b) = AT(A, b)\langle \sigma \rangle / 2 \tag{18}$$

is the dimensionless reduced thickness function and

$$x_{op} = \sigma_{op} / \langle \sigma \rangle \tag{19}$$

is the dimensionless reduced cross section operator. In other words, $\Gamma_i$ is determined by

$$\langle i \mid \exp \left(-t \, x_{op}\right) \mid i \rangle = \int dx f_i(x) \, \exp \left(-t \, x\right), \tag{20}$$

the generating function for the reduced distribution function $f_i(\mathrm{x})$.

The reduced thickness function $t(A,b)$ is expected to be largest for b=0 and to increase nearly monotonically with A. For the largest stable nuclei (A=238, say) , $t_{max} \approx 3.0$ for incident nucleons and 1.8 for pions. We can therefore not hope to get information on the generating function for $t$ greater than these values from nuclear total cross sections.

If $f_i(x)$ does not converge rapidly enough at large $x$ then the short-range and exponential approximations may be invalid. The short range approximation is essential if the nuclear cross sections are to be expressed in terms of $f_i(x)$, but the exponential approximation is to some extent just a convenience: a version of most of the results below could be obtained without it. For small t

$$\langle i \mid exp\ (-t\ x_{op}) \mid i \rangle\ =\ 1 - t + t^2 \langle i \mid x^2 \mid i \rangle\ /\ 2 - ...\ , \tag{21}$$

provided the series converges. (It does converge for all $t$ for all the examples of Refs. [1] and [2], but for the Poisson-like distributions the radius of convergence in n+1.) Since $\langle i \mid x^2 \mid i \rangle$ is about 1.4 to 1.5 for pions and 1.25 for nucleons, and some constraints on $\langle i \mid x^3 \mid i \rangle$ can be obtained from forward diffractive cross sections on deuterons, the generating function is already fairly well determined for small $t$. Nuclear cross sections will provide new information only if they can be used to constrain the generating function for values of $t$ larger than about 1.

The behavior of the generating function for very large $t$ is determined by the behavior of the distribution function near x=0:

$$\langle i \mid exp\ (-t\ x_{op}) \mid i \rangle\ \sim\ \frac{f_i(0)}{t}\ +\ \frac{f_i^{(1)}(0)}{t^2}\ +\ ...\ , \tag{22}$$

where $f_i^{(m)}(x)$ is the mth derivative of $f_i(x)$. If the generating function could be determined for large enough $t$, then, one could check the assumptions of Refs. [1] and [2] that $f_i(0)$ vanishes for nucleons, but not for pions. Unfortunately, as noted above, the range of $t$ is limited by the limit on the sizes of stable nuclei to values less than 3.

For many purposes it is useful to decompose Eq. (12), together with Eq. (17), as

$$\sigma_i(A)\ =\ \sigma_i^{(G)}(A) - \sigma_i^{(D)}(A), \tag{23}$$

7

where

$$\sigma_i^{(G)}(A) = 2 \int d^2b \; G^{(G)} \; (t(A,b)) \tag{24}$$

is the Glauber result, and

$$\sigma_i^{(D)}(A) = 2 \int d^2b \; G^{(D)} \; (t(A,b)). \tag{25}$$

is the decrease due to the dispersion in the hadron-nucleon total cross section. (It is easy to show that $G^{(D)}$ cannot be negative and vanishes only if there are no fluctuations in $\sigma$.) The integrand in Eq. (24) is simply

$$G^{(G)} \; (t) = 1 - exp \; (-t), \tag{26}$$

while

$$G^{(D)} \; (t) = \langle i \mid exp \; (-t \; x_{op}) \mid i \rangle - exp \; (-t). \tag{27}$$

Because $\sigma^{(D)}$ is a fairly small correction to $\sigma_i(A)$, in applying these formulas to experimental data it may be sufficient to include non-dispersive corrections, such a nuclear correlations and corrections to the short range approximation, only in $\sigma^{(G)}$, leaving $\sigma(D)$ as the simple expression above.

## IV. UNIFORM DENSITY LIMIT

As noted above, new information about the generating function can be obtained from total cross sections only for heavy nuclei. The nuclear densities $\rho(r)$ are then nearly constant in the nuclear interior, falling quite rapidly to zero at the nuclear surface. In this section the extreme uniform density limit is assumed:

$$\varrho(r) = \theta(R - r) \; / \; (4\pi R^3 \; / \; 3), \tag{28}$$

where $R \approx r_0 \; A^{1/3}$, with $r_0 \approx 1.1$ fm, is the nuclear radius. In this limit, replacing A by the equivalent and more convenient label R,

$$t(R,b) = t(R,0)\sqrt{1 - (b/R)^2}\ \theta\ (R - b), \tag{29}$$

where

$$t(R,0) \equiv \alpha R, \tag{30}$$

with

$$\alpha \equiv \sigma\ /\ (4\pi r_0^3\ /\ 3). \tag{31}$$

With this simple expression for $t(R,b)$, for any expression for a contribution to a total cross section of the form

$$\sigma_G = 2 \int d^2b\ G(t(R,b)) \tag{32}$$

it is easy to show that the derivative

$$d\sigma_G\ /\ (2\pi R^2) = G(t(R,0)). \tag{33}$$

In other words, the slope of the cross section contribution as a function of twice the nuclear geometric cross section is just the function G evaluated at the maximum value of $t(R,b)$ for that nucleus, obtained at zero impact parameter. Since G can be chosen to be 1 minus the generating function, or the generating function minus exp(-$t$), it is then trivial to extract the generating function at $t(R,0)$ from the derivative of the cross section. This relation, however, is exact only for the uniform density case. In the next section the accuracy of this relation for more realistic Woods-Saxon densities is studied.

## V. WOODS-SAXON DENSITIES

A more realistic nuclear density, especially for the heavy nuclei of main concern here, has the Woods-Saxon form

$$\varrho\ (R,r) = \frac{\varrho_0\ (R)}{1\ +\ exp\ ((r - R)\ /\ a_0)}, \tag{34}$$

9

where $a_0 \approx 0.523$ fm is the surface thickness parameter and

$$1 \,/\, \varrho_0 \,(R) \;\approx\; \frac{4}{3} \,\pi R^3 \,(1 \;+\; \pi^2 \,\frac{a_0}{R^2}) \tag{35}$$

is chosen so that the volume integral of $\rho$ is unity. (The parameter R for a number of different nuclei have been determined by fits to total cross sections at energies below 30 GeV/c, where inelastic contributions are expected to be small [6,7]. More exact nuclear densities can be determined from electromagnetic form factors, but the Woods-Saxon form is sufficient for present purposes.) Although there is an exact analytic expression for

$$t(R,0) \;=\; A\sigma\varrho_0 \;\{R \;+\; \ln \,[1 \;+\; exp \,(-R \,/\, a_0)]\}\,, \tag{36}$$

the thickness function for other values of b must be obtained by numerical integration. For this density the derivative relationship found in Sec. IV is not exact, although one might expect it to be a good approximation for large R where the density is quite uniform over most of the nucleus. In this section numerical results determining its accuracy are presented.

Fig. 1 shows $\sigma^{\mathrm{D}}(R)$ as a function of $2\pi R^2$ for 4 different pion-nucleon cross section distribution functions for pion-nucleus scattering. Distributions (b), (c), and (d) are the same as in Fig. 1 of Ref. [2], while distribution (a) is the superposition of two Poisson-like distributions introduced in Sec. II. Fig. 2 shows the analogous curves for nucleon-nucleus scattering, together with two experimental points, along with their error bars, from Ref. [6]. Here curve (a) is a two-term exponential while (b) and (c) are the n=2 and n=6 distributions shown in Fig. 4 of Ref. [1]. (The n=10 distribution gives results which are almost identical to those for n=6. In using the distributions from Refs. [1] and [2] we have adjusted the parameters slightly so that $\langle 1 \rangle \;=\; \langle x \rangle \;=\; 1.0$ more precisely.) For small $2\pi R^2$ all the distributions give nearly the same $\sigma^{\mathrm{D}}(R)$ because only low moments contribute, but for large $2\pi R^2$ there are differences of order 50 mb. Since these are of the same order as the experimental errors in Ref. [6], it is clear that more accurate experiments on heavy nuclei at very high energy are needed if the hadron-nucleus cross sections are to discriminate among different cross section distribution functions.

10

In Fig. 3 the slopes in Fig. 1 are compared with the $G^D(t(R,0))$'s for the various distributions. At large R the two quantities are similar, but $G^D$ is uniformly below the derivative, although the difference slowly decreases with increasing R. The corresponding results for nucleon-nucleus scattering are shown in Fig. 4. It seems that one might get a reasonable estimate for $G^D$ if $\sigma^D(R)$ could be measured accurately enough. This would be particularly true if an estimate for the surface corrections to the derivative relationship at large R could be found.

## VI. CONCLUSION

The results above suggest that accurate measurements of the total cross sections for the interactions of hadrons with a range of heavy nuclei at very high energies could increase our knowledge of the hadron-nucleon total cross section distribution functions, fixing the generating functions up to fairly large arguments and selecting among models suggested by theoretical prejudices and constraints on the first few moments. Very high energies are required for the validity of the "frozen configuration" approximation, while errors in measuring cross sections ranging to more than 3000 mb must be restricted to perhaps less than 10 mb. The dispersive effects are larger for pions than for nucleons, but unfortunately the maximum value of the argument of the generating function is smaller for pions because of their smaller cross sections on nucleons.

Even if the results of such difficult measurements were available, however, more work is required before they could be unambiguously interpreted in terms of the distribution functions. The formulas above depend upon a number of simplifying approximations: the frozen approximation, the neglect of nuclear correlations, the short range approximation, and the neglect of the imaginary part of the hadron-nucleon scattering amplitude. Fortunately, as noted in Ref. [6], the energy dependence of the total cross sections gives us a handle on many of these. In the range of energies high enough that the Glauber approximation is valid but low enough that inelastic intermediate states do not yet contribute significantly,

11

the nuclear total cross sections (as well as the differential cross sections) can be used to check assumptions about the nuclear wave functions. At the other extreme, the total cross sections at very high energies should approach constant values (aside from slow changes due to the energy dependence of the hadron-nucleon parameters) if the frozen approximation is valid.

It would be useful to have more information about the surface corrections to the relation between the generating function and the derivative of the total cross section as a function of R derived in Sec. IV in the uniform density approximation. Preliminary investigations suggest that for Woods-Saxon distributions these corrections should decrease as an inverse power of R. If this is true it should be possible to obtain more accurate values for the generating functions by studying the dependence of the total cross sections on R.

## ACKNOWLEDGMENTS

# REFERENCES

[1] B. Blättel, G. Baym, L.L. Frankfurt, H. Heiselberg, and M. Strikman, Phys. Rev. D **47**, 2761 (1993).

[2] B. Blättel, G. Baym, L.L. Frankfurt, H. Heiselberg, and M. Strikman, Phys. Rev. Lett. **70**, 896 (1993).

[3] B.K. Jennings and G.A. Miller, Phys. Rev. (to be published). Additional references to earlier work can be found here and in Refs. [1] and [2].

[4] M.L. Good and W.D. Walker, Phys. Rev. **120**, 1857 (1960).

[5] H.I. Miettinen and J. Pumplin, Phys. Rev. D **18**, 1696 (1978), and J. Pumplin, Physica Scripta **25**, 191 (1982).

[6] P.V.R. Murthy, C.A. Ayre, H.R. Gustafson, L.W. Jones, and M.J. Longo, Nucl. Phys. **B92**, 269 (1975).

[7] V. Franco, Phys. Rev. C **6**, 748 (1972).

FIGURES

FIG. 1.  Dispersive contribution to pion-nucleus scattering as a function of $2\pi R^2$, where R is the radius parameter in the Woods-Saxon nuclear density. Curves b, c, and d are calculated using the corresponding distribution functions shown in Fig. 1 of Ref. [2], while curve a is calculated using a distribution function which is a superposition of the n=0 and n=1 exponential distributions defined in Sec. II. This distribution was chosen to match distribution c at $\sigma$=0 and has $\langle \sigma^2 \rangle$ / $\langle \sigma \rangle^2$=1.7 instead of the value 1.5 of distributions b,c,and d.

FIG. 2.  Dispersive contribution to nucleon-nucleus scattering as a function of $2\pi R^2$, where R is the radius parameter in the Woods-Saxon nuclear density. Curves b, and c are calculated using the n=2 and n=6 distribution functions shown in Fig. 4 of Ref. [1], while curve a is calculated using a distribution function which is a superposition of n=1 and n=4 exponential distributions chosen to also give $\langle \sigma^2 \rangle$ = 1.25 $\langle \sigma \rangle^2$. All these distributions vanish at $\sigma$=0. The experimental points with their errors are taken from Ref. [6].

FIG. 3.  The solid curves are the derivatives of the corresponding curves in Fig. 1, while the adjacent dashed curves are the $G^{(D)}$ of Eq. (27) for the same distributions, evaluated at $t$(R,0) as given by Eq. (30).

FIG. 4.  The solid curves are the derivatives of the corresponding curves in Fig. 2, while the adjacent dashed curves are the $G^{(D)}$ of Eq. (27) for the same distributions, evaluated at $t$(R,0) as given by Eq. (30).

This figure "fig1-1.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9503006v1

This figure "fig2-1.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9503006v1

This figure "fig1-2.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9503006v1

This figure "fig1-3.png" is available in "png" format from:

http://arXiv.org/ps/nucl-th/9503006v1